# Coupled Magneto-Mechanical Analysis of Iron Sheets Under Biaxial Stress

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**Abstract— A directly coupled magneto-mechanical model is used to analyse the coupled behavior of iron sheets under biaxial magneto-mechanical loading applied by a novel biaxial single sheet tester device. Magneto-mechanically coupled constitutive equations of the material derived using an energy based approach are integrated into a finite element model of the single sheet tester device and simulations are performed to solve for the displacement fields and the magnetic vector potential in the material. The obtained magnetostrictive strain curves of the studied material under different magneto-mechanical loadings are presented.**

***Index Terms*— Finite element analysis, Helmholtz free energy, magnetic field induced strain, magneto-elasticity, magneto-mechanical effects, magnetostriction.**

## I. INTRODUCTION

MAGNETOSTRICTION is a phenomenon in ferromagnetic materials which causes strain in the presence of magnetic fields. This effect is known to be the source of acoustic noise in transformers, electrical machines and various inductive components. For instance, in [1]-[2] it is shown for transformers and electrical machines that magnetostriction clearly causes noise. In [3] it is reported that the magnetostriction is the main cause for noise in pulse width modulation operated inductive components in the medium frequency range.

Since magnetostriction depends on the stress and magnetic field it is a coupled magneto-mechanical problem. These problems can be solved with indirect or direct coupled approaches [4]-[6]. In an indirect coupled analysis, new magnetic input quantities are evaluated from the previous mechanical output quantities. On the other hand, in the direct coupled problems, magnetic and mechanical quantities are solved simultaneously meaning that mutual dependency is taken into account. In both methods some measurements often are made to obtain necessary material data. For instance, in [6] the magnetic field is calculated using beforehand measured stress dependent permeability curves and in [7] stress-dependent magnetostriction curves are obtained experimentally to calculate the permeability variation.

In this paper a coupled magneto-mechanical model of a novel biaxial single sheet tester device is developed using a directly coupled magneto-mechanical model for magneto-elastic deformation. The implementation of the model to the finite element method (FEM) is performed using Comsol Multiphysics® and Livelink for Matlab® interface in order to analyse and characterize the coupled magneto-mechanical properties of the studied material.

## II. METHOD

Based on [8] constitutive equations for coupling the magnetic and elastic properties of the material are derived from Helmholtz free energy density $\psi$ . In case of a magneto-elastic material this energy is a function of magnetic flux density $\boldsymbol{B}$ and strain tensor $\boldsymbol{\varepsilon}$ and can be expressed by using the following five invariants as $\psi(I_1,I_2,I_4,I_5,I_6)$ . Where $I_1,I_2,I_3,I_4,I_5,I_6$ are:

$$I_1 = \mathrm{tr}\boldsymbol{\varepsilon},\ \ I_2 = \frac{1}{2}\mathrm{tr}\boldsymbol{\varepsilon}^2,\ \ I_3 = \frac{1}{3}\mathrm{tr}\boldsymbol{\varepsilon}^3$$
$$I_4 = \boldsymbol{B}\cdot\boldsymbol{B},\ \ I_5 = \boldsymbol{B}\cdot(\boldsymbol{\varepsilon}\boldsymbol{B}),\ \ I_6 = \boldsymbol{B}\cdot(\boldsymbol{\varepsilon}^2\boldsymbol{B}). \tag{1}$$

Since linear elastic material is assumed $\psi$ does not depend on the third invariant $I_3$ .

Using the Helmholtz free energy density and the chain rule of derivative, the constitutive equations for the Cauchy stress tensor $\boldsymbol{\sigma}$ and the magnetization vector $\boldsymbol{M}$ can be expressed as

$$\boldsymbol{\sigma} = \rho\sum_{i=1,i\neq 3}^{6}\frac{\partial\psi}{\partial I_i}\frac{\partial I_i}{\partial\boldsymbol{\varepsilon}} \ \text{ and } \ \boldsymbol{M} = -\rho\sum_{i=1,i\neq 3}^{6}\frac{\partial\psi}{\partial I_i}\frac{\partial I_i}{\partial\boldsymbol{B}}. \tag{2}$$

The magnetic field strength vector $\boldsymbol{H}$ depends on $\boldsymbol{B}$ as $\boldsymbol{H} = \nu_0\boldsymbol{B} - \boldsymbol{M}$ . Here $\nu_0$ is the reluctivity of vacuum. Total electromagnetic forces acting on the iron are known as electromagnetic stress tensor $\boldsymbol{\tau}_{\mathrm{em}}$ and given by

$$\boldsymbol{\tau}_{\mathrm{em}} = \nu_0\boldsymbol{B}\boldsymbol{B} - \boldsymbol{B}\boldsymbol{M} - \frac{1}{2}\nu_0(\boldsymbol{B}\cdot\boldsymbol{B})\boldsymbol{I} + (\boldsymbol{B}\cdot\boldsymbol{M})\boldsymbol{I}. \tag{3}$$

where $\boldsymbol{I}$ is the identity tensor. The total stress tensor $\boldsymbol{\tau}$ is expressed as the sum of $\boldsymbol{\tau}_{\mathrm{em}}$ and the Cauchy stress tensor $\boldsymbol{\sigma}$ as $\boldsymbol{\tau} = \boldsymbol{\sigma} + \boldsymbol{\tau}_{\mathrm{em}}$ . Using the balance equations for magneto-elastic solids and taking into account external mechanical forces $\boldsymbol{f}_{\mathrm{mec}}$ we obtain

$$-\nabla\cdot\boldsymbol{\tau} = \boldsymbol{f}_{\mathrm{mec}}. \tag{4}$$

On the other hand, assuming that there are no source currents the Ampere's law in iron is

$$\nabla\times\boldsymbol{H} = 0. \tag{5}$$

The magneto-elastic problem studied is nonlinear. Therefore, Newton-Raphson method is used to solve these coupled constitutive equations. The nodal values of the magnetic vector potential and the displacements are solved using FEM. The strain and the magnetic flux density are then obtained as

$$\boldsymbol{\varepsilon} = \frac{1}{2}(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^{\mathrm{T}}) \qquad (6)$$
$$\boldsymbol{B} = \nabla \times \boldsymbol{A}.$$

where $\nabla \boldsymbol{u}$ is the displacement gradient and $\boldsymbol{A}$ is the magnetic vector potential.

## III. RESULTS AND DISCUSSIONS

The parameters needed to model the properties of the studied material, 0.5 mm Si-Fe sheets, were determined using experimental data obtained from a uniaxial single sheet tester. In the experiment process the material was loaded with different stresses from 50 MPa compression (–) to 80 MPa tension (+) parallel to the flux density and magnetization curves were measured. Afterwards, initial fitting of the single valued model parameters to the H-averaged measured magnetization loops was realized at various stress values as shown in Fig. 1(a). Magnetic materials show reduced permeability under high compression and tension. This behavior of the magnetic materials is modeled successfully using the model. Besides, increased permeability is observed under low tensile stress as reported in [9].

In Fig. 1(b) magnetostrictive strain which was calculated using the model is presented. These results were obtained by integrating the model to the finite element analysis (FEA) software Comsol Multiphysics® using Livelink for Matlab® interface. FEA was performed for magnetic vector potential and the displacements of the nodes.

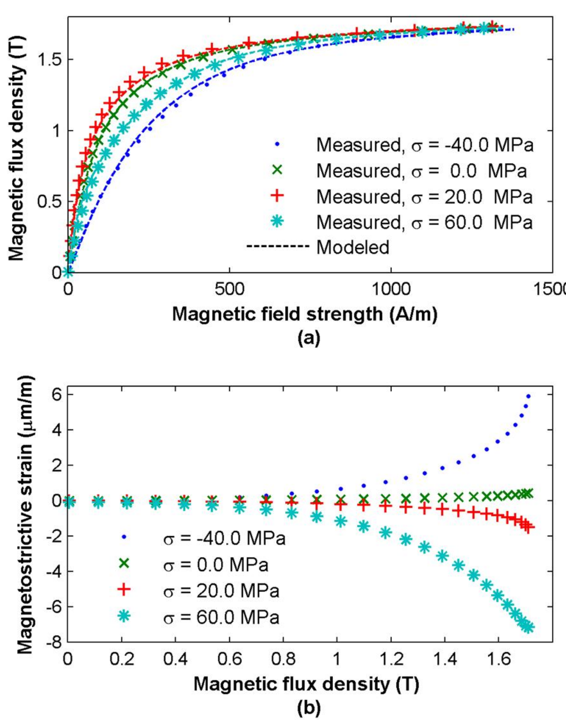


Fig. 1. Model results using various tensile and compressive stress values. (a) Fitting results of single valued model parameters to H-averaged magnetization curve measurements. (b) Calculated unidirectional magnetostrictive strains under different magneto-mechanical loading conditions.

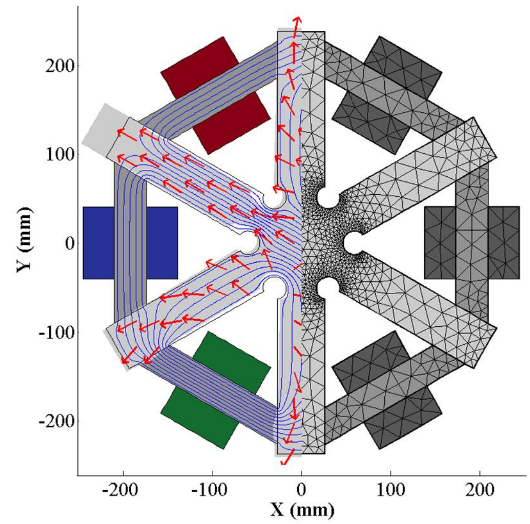


Fig. 2. Biaxial single sheet tester design, deformed sample and meshed geometry.

In the full paper magneto-mechanical analysis of iron sheets in a novel biaxial single sheet tester design using the presented model will be given. The sample sheet is designed to be able to have uniform flux density and arbitrary stress distribution in the central area. This will be done by magnetizing the sample by six coils which are placed around the magnetizing yoke and mechanically loading the six arms of the sample in pairs. The design of the device is shown in Fig. 2. In the left side of Fig. 2 the magnetized sample without any mechanical loading is shown. Here, arrows represent the deformation of the sample body due to magnetostriction. On the right side of the Fig. 2 the meshed device geometry is shown. Both alternating and rotating flux conditions will be studied in the full paper.

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